Impedance Matching in High-Power Microwave Applications

Notes to Bilik-ImpiWebin191024.pptx

Vladimir Bilik

Table of Contents

1. Introduction

Impedance matching is a subject concerned with maximizing the power transferred from a generator to a load. It is significant in several areas of electrical engineering. In communications, it pertains to conveying information, in particular

- recovering weak signals from noise,
- undistorted transmission of pulses; for instance, in digital communications and radar.

In high power microwave energy applications, impedance matching is connected with a different goal: an efficient use of the energy produced by a source of energy (a generator).

Although the theoretical foundations for these objectives are the same, the specific requirements may widely differ, resulting in very distinct implementations of matching devices and procedures.

This document will focus on impedance matching in high power microwave engineering. Nevertheless, we will start with a review of the basics of the impedance matching theory in general.

2. Outline (Slide 2)

- Basic theory
- Impedance matching in industrial applications
- Example: single-stub matching
- Automatic impedance matching
- Optimal matching
- Autotuner example

3. Basic Theory

3.1 Definition (Slide 4)

The basic components of an industrial microwave system include:

- Generator
- Load (applicator)
- Interconnecting transmission line (waveguide, coaxial line, etc.)

Impedance matching means maximizing the power P_L delivered from a generator to a load. The d*elivered* power denotes the power *absorbed* in the load (for instance, converted to heat).

The matching is accomplished by manipulating the reflection coefficient Γ_1 presented to the generator. This can be done by means of inserting a *matching network* between the generator and the load.

3.2 Components Characterization

3.2.1 Transmission Line (Slide 5)

The transmission line can be typically a waveguide or a TEM line, such as a coaxial line (coax). The main characteristics are the characteristic impedance Z_0, the guide wavelength λ_g, and length L.

We will neglect the line attenuation. We will also suppose that only the dominant mode of the electromagnetic waves can propagate in the line.

Characteristic Impedance

The characteristic impedance Z_0 of a transmission line can be uniquely defined only for TEM lines. In waveguides, the definition of impedance is ambiguous. However, we will, in general, make use of the *normalized* impedances $z = Z/Z_0$ and normalized admittances $y = 1/z$, which can be used generally. In this context, we can deem the normalized characteristic impedance of the interconnecting transmission line $z_0 = 1$.

Guide Wavelength

The guide wavelength λ_g is, in general, given by the formula

$$(1) \qquad \lambda_g(f) = \frac{c_0}{\sqrt{f^2 - f_c^2}}$$

where c_0 is the speed of light in free space and f_c is the waveguide cutoff frequency ($f_c = 0$ for TEM lines).

Length

The physical length L of a transmission line section can be alternatively expressed in terms of the *electric length* θ in radians or degrees where

(2) $$\theta_{rad} = 2\pi \frac{L}{\lambda_g}, \qquad \theta_{\deg} = 360 \frac{L}{\lambda_g}$$

Waves

A wave travelling in a transmission line can be characterized by a variable called the *complex amplitude*

(3) $$a(x) = Ae^{j\varphi(x)}$$

where x is the coordinate along the transmission line. The wave variable magnitude A is derived from the power P transmitted by the physical wave. Similarly to voltage waves in a TEM line, A is proportional to the square root of the transmitted power:

(4) $$A = |a| = \sqrt{P}$$

The phase $\varphi(x)$ is equal to the phase of the physical wave.

Two waves can generally travel in opposite directions along the line:

- Forward wave (left to right), characterized by its complex amplitude $a(x)$
- Reverse wave (right to left), characterized by the complex amplitude $b(x)$

Transmission Line as a Reference for Reflection Coefficient Definition

The interconnecting transmission line serves as a reference for defining the scattering parameters and reflection coefficients in the following sense:

Supposing that the forward wave a is the excitation stemming from the generator, and it is incident on the load, then the reverse wave b is due to the reflection from the load. The reflection coefficient at a plane x is then defined as

(5) $$\Gamma(x) = \frac{b(x)}{a(x)}$$

As we will see later, the other scattering parameters can also be defined in a similar manner.

Normalized Impedance and Admittance

The normalized impedance z and admittance y are related to the reflection coefficient as

(6) $$z = \frac{1}{y} = \frac{1+\Gamma}{1-\Gamma}$$

Consequently, there is one-to-one transform between Γ and z or y. Therefore, when we speak about the reflection coefficient, impedance or admittance, we are in fact speaking about the same entity.

3.2.2 Load (Slide 6)

The load is fully characterized by its reflection coefficient

(7) $$\Gamma_L = b/a$$

all quantities being defined with respect to a given reference plane T

If we say that the load is *matched*, this means that the load does not give rise to the reverse wave in the interconnecting line, and therefore $\Gamma_L = b/a = 0$.

In this sense, the “match” of a load refers to the transmission line to which it is connected.

3.2.3 Generator

The generator is a source of harmonic (sinusoidal) electromagnetic field. The generator will be characterized by:

- The generated frequency f
- The power P_G delivered to a nonreflecting load (we will also call it *generator power*)
- The internal reflection coefficient Γ_G, defined at a given reference plane

Γ_G is observed when the driving signal (P_G) is set to zero. Then the generator acts as just a load.

If we say that the generator is *matched*, this means that it does not reflect the external wave incident upon it. Thus, for a matched generator, $\Gamma_G = 0$.

3.2.4 Two-Port Network (Slide 7)

A two-port network distributes (scatters) incoming waves (excitations) a_1 and a_2 to both ports, resulting in the outgoing waves (responses) b_1 and b_2. The process is characterized by the equations

(8) $$b_1 = S_{11}a_1 + S_{12}a_2$$

(9) $$b_2 = S_{21}a_1 + S_{22}a_2$$

These relations are often expressed in the form of a signal flowgraph. The four complex proportionality coefficients, the *scattering parameters* (S-parameters)

S_{ij} $i = 1, 2,$ $j = 1, 2$

fully characterize the two-port.

The parameters can be arranged to a square 2 × 2 matrix (scattering matrix or S-matrix).

Since the scattering parameters relate the waves in the attached transmission lines, the S-parameters are always referred to these attached lines.

If only port 1 is excited ($a_1 \neq 0$) and port 2 is terminated in a match (and hence $a_2 = 0$), the equations reduce to

(10) $$b_1 = S_{11}a_1$$

(11) $$b_2 = S_{21}a_1$$

Consequently, S_{11} can be interpreted as the input reflection coefficient Γ_1 of the two-port provided port 2 is terminated in a match. Similarly, S_{21} can be interpreted as the transmission coefficient t_{21} of the two-port provided port 2 is terminated in a match. The analogous is true for the reverse direction.

3.3 Matching Network (Slide 8)

The matching network is a fixed or adjustable (usually lossless) two-port network that is said to be transforming the load reflection coefficient Γ_L to the input reflection coefficient Γ_1. Such matching networks are therefore also called *impedance transformers*. The $\Gamma_L \rightarrow \Gamma_1$ transformation is governed by the equation

(12) $$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

When the matching network is lossless (which we will assume throughout), the power P_1 accepted by the network at its input is equal to the power P_L delivered to the load. The power P_1 can be maximized, and hence the impedance match can be achieved, by appropriately adjusting the network scattering parameters S_{ij}.

3.4 Matching Condition (Slide 9)

The analysis of the simple equivalent circuit shows that power P_1 accepted by the matching network, and hence power P_L delivered to the load, is

(13) $$P_1 = P_G \frac{1-|\Gamma_1|^2}{|1-\Gamma_G\Gamma_1|^2}$$

The amount of P_1 is the result of the interplay between the reflection coefficients Γ_1 and Γ_G. The question now arises: Given the generator reflection coefficient Γ_G, for what value of Γ_1 will the power P_1 be maximized? Mathematical analysis of (13)[1] leads to the condition

(14) $$\Gamma_1 = \Gamma_G^*$$

The condition states that in order to achieve the maximum delivered power, the reflection coefficient Γ_1, perceived by the generator, must be the complex conjugate of the generator reflection coefficient Γ_G. The delivered power is then

(15) $$P_{1\max} = \frac{P_G}{1-|\Gamma_G^2|}$$

If the load seen by the generator is matched ($\Gamma_1 = 0$), the delivered power is equal to P_G, which is actually the definition of P_G.

3.5 Matching Rule (a little) Generalized (Slide 10)

The matching condition (14), formulated for the matching network input, can be generalized for other locations along the signal path. We can call it "Look Left – Look Right" principle, and it can be articulated as follows:

> The state of the impedance match is achieved if at any port-type cross section x along the signal path the reflection coefficients seen to the left (toward generator) and to the right (toward load) are mutually conjugated.

A "port-type" cross section is, roughly speaking, a cross section through which the complete signal passes and where such reflection coefficient can be defined (i.e. measured). We can, for instance, be able to bisect the waveguide of a waveguide-type matching network so that a part of it is on the generator side (left), and the other part is on the load side (right). Looking left, we see the reflection coefficient Γ_{Gx}, looking right we see the reflection coefficient Γ_{Lx}. The whole network on the left can then be considered an equivalent generator with internal reflection coefficient Γ_{Gx}, and the whole network on the right can be treated as an equivalent load with reflection coefficient Γ_{Lx}. The matching condition therefore assumes the generalized form

(16) $$\Gamma_{Gx} = \Gamma_{Lx}^*$$

When applied at the matching network output, the condition has the form

(17) $$\Gamma_2 = \Gamma_L^*$$

where Γ_2 is the output reflection coefficient of the matching network.

3.6 Matched Generator (Slide 11)

For the important case of a matched generator ($\Gamma_G = 0$), the delivered power formula (13) reduces to the well-known difference between the incident power P_i and the reflected power P_r:

(18) $$P_1 = P_G\left(1-|\Gamma_1|^2\right) = P_G - P_G|\Gamma_1|^2 = P_i - P_r$$

where the reflected power is

(19) $$\Gamma_1 = \Gamma_G^*$$

---

[1] The analysis is straightforward, although tedious, and is better carried out in terms of normalized impedances.

The matching condition (14) becomes

$$\Gamma_1 = 0 \quad (20)$$

Hence, for a matched generator, the load to achieve the maximal delivered power must also be matched. The delivered power is then, by definition, $P_{1max} = P_G$.

3.7 Impedance Matching Summary (Slide 12)

The many variants of the matching condition are summarized in Tab. 1. For the case of a matched generator, the output condition (17) reduces to

$$S_{22} = \Gamma_L^* \quad (21)$$

This is due to the fact that a matched generator acts as a matched termination to port 1. Therefore, by definition, the output reflection coefficient $\Gamma_2 = S_{22}$ of the matching network.

Tab. 1. Summary of the matching conditions for the matching network input and output.

Generator	Input	Output
$\Gamma_G \neq 0$	$\Gamma_1 = \Gamma_G^*$	$\Gamma_2 = \Gamma_L^*$
$\Gamma_G = 0$	$\Gamma_1 = 0$	$S_{22} = \Gamma_L^*$

The input condition is often useful to be employed for the following tasks:

- Design of components
- Matching of constant loads

The output condition may be useful for automatic matching of variable loads. Here, the procedure runs as follows: For a given Γ_L, adjust the tuner until its $S_{22} = \Gamma_L^*$.

3.8 Magnetron Generators (Slide 13)

In this context, by "magnetron", we actually refer to a magnetron head, i.e. a magnetron installed in a launcher. The reflection coefficients (Γ_G, Γ_1) are then defined at the launcher output, or at the reference plane of the magnetron Rieke diagram.

3.8.1 Magnetrons with Circulators

Magnetrons with circulators are matched because the reverse wave is absorbed in the circulator's waterload. The matching criteria for the case $\Gamma_G = 0$ therefore do apply.

3.8.2 Magnetrons without Circulators

Magnetrons without circulators, e.g. those in domestic microwave ovens are **far from matched**. But they are designed to perform best if their load reflection coefficient is matched or nearly matched: $\Gamma_1 \approx 0$. Therefore, the criteria for the matched generator case **still apply**.

3.9 Solid State Generators (Slide 14)

In the emerging solid state generators, the output medium is typically a coaxial line with characteristic impedance $Z_0 = 50$ ohms, and the generator impedance is also approximately so: $Z_G \approx 50$ ohms. The matching criteria for the case $\Gamma_G = 0$ therefore also apply.

3.10 Matching Bandwidth (Slide 15)

The matching bandwidth is determined by the frequency dependence of the input reflection coefficient magnitude $|\Gamma_1(f)|$.

In the theory so far, we implicitly assumed matching at a single frequency, which enabled us to achieve a perfect match at that single frequency. This is often sufficient in the field of high-power microwave energy applications because the ISM bands are narrow (2 – 4%). However, it may not always be sufficient. For instance:

- Components must work in a wider bandwidth. As an example, waveguide-to-coax adapters often serve for wideband overview measurements of microwave applicators, and must therefore cover much wider range than merely an ISM band.

- Applicator design matching band must be wide enough to accommodate:
  - Manufacturing tolerances
  - Materials tolerances
  - Generator frequency variations
  - Temperature variations, etc.

Matching over wider frequency range must at least be examined if not specifically aimed at.

3.11 Matching over a Frequency Range (Slide 16)

Note: The information for this slide has been predominantly gained from the book [1]. For the sake of clarity and general understanding, the explanation here is simplified and not all the affirmations are rigorously true. For rigorous treatment, see the references in [1] to the original works of Bode and Fano.

The buzzword for the matching over a definite frequency range reads: *Resources are limited*. The "resources" in this context is the area A_L between the curve of $|\Gamma_L(f)|$ expressed in dB,

$$m = 20\log\left|\Gamma_L\left(f\right)\right| \tag{22}$$

and the horizontal line corresponding to the value 0 dB, representing a total reflection. Here Γ_L is the load reflection coefficient. Mathematically

$$A_L = -\int_0^\infty 20\log\left|\Gamma_L\left(f\right)\right| df \tag{23}$$

The theory argues that for any matching network, the *input* area

$$A_1 = -\int_0^\infty 20\log\left|\Gamma_1\left(f\right)\right| df \le A_L \tag{24}$$

can never exceed that of the load area A_L. The consequence is that there is a tradeoff between matching bandwidth, and best possible degree of impedance match in that band, expressed in dB as $m_{\max}$.

We can, for instance alternatively:

- Minimize the mismatch in a wider band $f_a - f_b$ to a certain worst value m_1, or
- Minimize the mismatch in a narrower band $f_1 - f_2$ to a lower worst value m_2.

In both cases the "resource" areas are ideally equal: $A_1 = A_2 = A_L$.

This is the best possible case: we can, of course, "botch" the matching network design and "achieve" the matching with values of A_1 or A_2 much lower than A_L.

Obviously, the ideal case is the rectangular shape of $|\Gamma_1(f)|$, for which all the resource area is "stuffed" into the band of interest and nothing is left out of the band (i.e., the matching network behaves as a total reflection). The rectangular shape of $|\Gamma_1(f)|$, however, represents an *ideal filter*. Consequently, matching circuit design is very much a *filter* design. And the filter design is a highly developed and sophisticated branch of science. Fortunately, in the area of the high-power microwave energy applications, we will only seldom need it. Instead, the typical process is as follows:

1. We pick a given (simple) matching structure. This structure has a definite shape of the frequency response $\Gamma_1(f)$ and we will have little control over it.
2. We will try to achieve a best possible degree of impedance matching by minimizing $|\Gamma_1(f)|$ at a desired frequency (typically a center of an ISM band, or the magnetron nominal frequency).
3. Better yet, we will try to achieve an equal mismatch at the edges of a desired band (e.g. 2425 – 2475 MHz).

3.11.1 Matching of High Reflection Coefficients

This subject was not part of the webinar presentation.

High load reflection coefficients can theoretically also be matched. However, due to the high value of $|\Gamma_L|$, the resource area A_L is small. This has the following consequences:

- The bandwidth of an acceptable match is very narrow.

- The match is very sensitive to the matching network variables.
- The matching is very sensitive to the load variations.
- The matching structure contains localities with very high field strengths and surface currents, which leads to the risk of arcing or overheating.

The applicators should possibly be designed such that $|\Gamma_L|$ be not extremely high, even if the use of an automatic impedance matching device (autotuner) is planned. The practical limit value is $|\Gamma_L| \approx 0.8$. In some cases, this is not possible, such as in sintering of ceramic powders, which usually have very low loss tangent at the starting low temperatures. The matching network in this case typically consists of a cavity resonator with the material sample inside, the resonator being loosely coupled to the generator. The matching response $|\Gamma_1(f)|$ has the form of a very narrow resonance curve with the minimum at the loaded cavity resonance frequency f_r. The resonance frequency can easily differ from the generator frequency f_G, making the load at f_G practically a total reflection, $|\Gamma_1(f_G)| \approx 1$. In such a case an autotuner will also not help. Therefore, either the resonator or the generator must include a means of tuning for aligning f_r with f_G.

3.12 Filter Bible (Slide 17)

The book [1], also known as the *Filter Bible*, is an excellent and highly esteemed source of knowledge for those who would like to go deeper into the theory and practice of filters and impedance matching networks. It explains basic concepts, provides an abundance of supporting references for each topic, and gives a wealth of practical examples, starting from simple structures to very complex ones, with detailed description of design procedure and many practical tips.

4. Impedance Matching in High-Power Applications

4.1 Matching Tasks (Slide 19)

Matching tasks in high-power applications can be divided into three main groups:

1. Design of components
2. Matching applicators with constant loads
3. Matching applicators with variable loads

4.1.1 Design of Components

By components we mean parts with universal use, such as waveguide transitions, waveguide-to-coax adapters, waveguide bends, waveguide windows, power splitters, directional couplers, circulators, etc. Their main features are relatively wide bandwidths, and that their design supposes matched terminations as loads.

The design methods often do not follow the strictly established filter design principles but can be described as “speculative”, involving physical understanding of the problem, experience, intuition, and some amount of trial-and-error. Electromagnetic simulation is an indispensable design tool.

Application engineers only rarely must resort to components design. Instead, they usually buy components available from manufacturers, who do the design job for them.

4.1.2 Applicators with Known, Essentially Constant Loads

The load for this applicator class is mismatched, i.e. $\Gamma_L(f) \neq 0$, but steady or only slightly varying. A fixed matching circuit, adjusted once for all, is the appropriate matching solution. The design methods include use of a Smith chart, simple analytic formulas, equivalent circuits, etc. Circuit or electromagnetic simulators represent a significant aid in the design procedure.

4.2 Applicators with Variable Loads (Slide 20)

The load reflection coefficient $\Gamma_L = \Gamma_L(f, t)$ for this applicator class is mismatched *and* variable, often in a wide range. This is a common scenario in the industry. An adjustable matching network, called a *tuner*, is the appropriate matching solution. Two cases are distinguished:

- If the load varies only occasionally (e.g. a product changes from time to time):
  - A *manual* tuner is adequate.
  - No qualification is needed for adjusting: the operator just "fiddles" with the tuner controls in a trial-and-error way until an acceptable matching is reached.
- If the load varies often or continuously:

– An *automatic* impedance matching device (the autotuner) should be used.
– Although the autotuner works automatically, it still must be configured appropriately for given circumstances (typically dictated by the used magnetron power supply and the nature of the load reflection coefficient variations).
– Operating personnel must therefore understand not only workings of their industrial system but also basics of the autotuner operation (having a kind of "tuner driving license").

4.3 Specifics of Matching in High-Power Applications (Slides 22, 23)

Item	Common Electronics	Hi-Power Applications
Objective	Conveying of information • Retrieving weak signals from noise • Transmission of pulses without distortion	Delivering power
Power	Low: 1 μW – 10 W	High: 1 – 100 kW
Center frequency	Known, constant	Unknown, variable
Signal spectrum	Wide: $n \times 10\%$	Very narrow: 0.01 – 0.1%
Matching bandwidth	Wide: as that of signal	Narrow: 2%
Matching network	Lumped	Waveguide structure
Complexity	Complex: • Very stringent demands	**Simple**
Network losses	Acceptable (not always)	Unacceptable!
Field strength	Not of concern	Much of concern
Load	• Frequency-dependent • Often mismatched • Time-independent	• Frequency-dependent • Mismatched • Time-variable
Generator	As load	Matched
Adjustability	No • Fixed matching network	Yes • Manual • Automatic

4.4 Summary: High-Power Matching Features (Slide 24)

Typical features of matching in the high-power microwave energy applications are:

- Matching networks are typically waveguide-based structures.
- Matching networks are narrowband. This implies that:
  – The matching structure is simple.
  – Design for a single frequency is acceptable: In most cases, the matching bandwidth will be satisfactory.
- Matching networks can be:
  – Fixed
  – Adjustable: either
    - manually (tuners), or
    - automatically (autotuners).

5. Example: Single-Stub Matching

5.1 Problem (Slide 26)

The problem configuration is as follows:

- Cubic cavity applicator, sides 100 mm long.
- The applicator is fed from a common WR340 waveguide (86.36 mm × 43.18 mm). The load reflection coefficient reference plane T is spaced 20 mm from the inner surface of the cavity wall.
- Working frequency: 2.45 GHz.
- Load: A cup of water in the center, implemented as a cylinder with both diameter and height 50 mm. The water volume is approximately 100 ml.

The used permittivity model approximates water complex relative permittivity $\varepsilon_r = \varepsilon' - j\varepsilon''$ by the formulas

(25) $$\varepsilon' = 84.17 - 0.27483\,T - 3.3526\cdot 10^{-4}\,T^2$$

(26) $$\varepsilon'' = 22.02 - 0.61682\,T + 7.0286\cdot 10^{-3}\,T^2 - 2.8494\cdot 10^{-5}\,T^3$$

where T is the water temperature in Celsius. We will consider T = 30°C, resulting in ε' = 75.6, ε'' = 9.07.

5.2 Load Reflection Coefficient (Slide 27)

The load reflection coefficient

(27) $$\Gamma_L = M\,\mathrm{e}^{j\varphi_L}$$

at plane T in the frequency range 2.2 – 2.7 GHz was obtained using the CST Studio Suite electromagnetic simulator. At 2.45 GHz, M = 0.4845, φ_L = 201.9°.

In practice, results of this type can often be obtained via measurement from a vector network analyzer (VNA).

The goal is to improve the match at 2.45 GHz, or, better yet, in the 50-MHz band spanning the frequencies 2.425 – 2.475 GHz.

5.3 Single-Stub Matching (Slide 28)

The single-stub impedance matching consists in introducing a suitable parallel susceptance b_B at a suitable distance d_A from the load reference plane T.

Let the plane, located at the distance d_A from T, be designated plane A. Let the normalized admittance seen looking into the applicator at plane A be y_A.

- The suitable distance d_A is where admittance y_A has unit real part, i.e.

(28) $$y_A = 1 + jb_A$$

- The suitable susceptance b_B is that which *cancels out* the imaginary part jb_A of admittance y_A, i.e. $b_B = -b_A$.

The result is a match because the resulting normalized admittance is

(29) $$y_1 = y_A + jb_B = 1 + jb_A + jb_B = 1$$

The compensating susceptance b_B will be realized by a tuning stub, which introduces a *positive* (capacitive) susceptance type $b_B > 0$. Because of this, distance d_A must be such that b_A is negative: $b_A = -|b_A|$.

5.4 Principle (Slide 29)

When we add a section of transmission line with length d_A to a load with reflection coefficient Γ_L, the line input reflection coefficient at plane A

(30) $$\Gamma_A = \Gamma_L\,\mathrm{e}^{j\Delta\varphi}$$

will be equal to Γ_L rotated clockwise by an angle (in degrees)

(31) $$\Delta\varphi = 720\frac{d_A}{\lambda_g}$$

To satisfy the condition (28) of a unit real part of y_A, Γ_A must lie on the circle K_1 in the admittance Smith chart. The susceptance b_A will be negative on the upper half of K_1.

To compensate for b_A, the susceptance added by the stub must be positive: $b_B = +|b_A|$.

5.5 Determining Stub Distance (Slide 30)

For our particular case $\Gamma_L = 0.4845 \angle 201.9°$ at 2.45 GHz, the rotation angle is $\Delta\varphi = 83°$. Using (31), this corresponds to the waveguide length d_A = 19.97 mm. In fact, we can, in general, use distances

(32) d_A = 19.97 mm + $n \times \lambda_g/2$ = 19.97 mm + n × 86.69 mm, where $n = 0, \pm 1, \pm 2, \ldots$

This is because adding a multiple of the guide half-wavelength $\lambda_g/2$ only rotates the point in the Smith chart by 360°.

The presented geometrical solution can in fact be replaced by the following sequence of simple Excel-programmable equations. Supposing

(33) $$\Gamma_L = M \exp\left(j\varphi_L\right)$$

then

(34) $$\Gamma_A = M \exp\left(j\varphi_A\right)$$

where, using the Excel atan2 function,

(35) $$\varphi_A = \text{atan2}\left(-M^2, M\sqrt{1-M^2}\right)$$

The rotation angle in radians

(36) $$\Delta\varphi = \varphi_L - \varphi_A$$

must be wrapped to the interval $\langle 0; 2\pi)$. Then

(37) $$d_A = \frac{\Delta\varphi}{2\pi}\frac{\lambda_g}{2} + n\frac{\lambda_g}{2} \quad n = 0, \pm 1, \pm 2, \ldots$$

5.6 Determining Stub Length (Slide 31)

We will now show that in order to determine the stub length we do not actually need to know neither the susceptance b_A, nor the dependence of stub susceptance b_B on the stub length h; we only need to know stub S_{11} as a function of h.

The point B in the admittance Smith chart, which is the complex conjugate of the reflection coefficient Γ_A, (remember the look left – look right principle) represents the admittance

(38) $$y_B = 1 + jb_B$$

where b_B is the stub susceptance. The formula implies that the point B is actually the scattering parameter S_{11} = S_{22} of the stub. And because B ≡ S_{11} lies on the circle K_L, the identity is valid

(39) $$|S_{11}| = |\Gamma_L| = M$$

Instead of determining susceptances, to find the stub length h we only have to solve the equation

(40) $$|S_{11}(h)| = |\Gamma_L|$$

for the unknown h. For this, we will need to know the dependence of the stub S_{11} on its length. Out next immediate task is to find this dependence.

5.7 S_{11} of Tuning Stub (Slide 32)

The tuning stub used for the matching has a diameter of 20 mm and its tip is rounded with a radius of 7 mm. We obtained the dependence of the stub's S_{11} on its length h by electromagnetic simulation in 2.2 – 2.7 GHz range (for such a simple structure, the measurement and simulation agree very well). A graphical form of this dependence for 2.45 GHz is shown as the blue curve. For matching, the inverse function (red curve)

$$h = f\left(\left|S_{11}\right|\right) \tag{41}$$

is more useful because it can be readily approximated by a 3rd degree polynomial

$$h = 3.341 + 44.278\left|S_{11}\right| - 53.402\left|S_{11}\right|^2 + 37.793\left|S_{11}\right|^3 \tag{42}$$

with h in mm. For $|S_{11}| = |\Gamma_L| = 0.4845$, the polynomial yields $h = 16.56$ mm.

It seems that we have now completed the matching task, because we have arrived at both d_A and h. However, there is a minor problem.

5.8 Slight Hitch (Slide 33)

The problem consists of the fact that a thick stub does not exactly behave as a mere parallel susceptance. It is demonstrated by the fact that the contour of $S_{11}(h)$ for a fixed frequency does not follow the unit real part circle K_1 in the admittance Smith chart. For $f = 2.45$ GHz, the fit is much better if the whole contour is rotated clockwise by about 7.5°. This fact can be accounted for by the stub susceptance b being embedded in a section of waveguide with certain length d_0 at both sides. Strangely, the equivalent length appears to be negative: $d_0 = -1.8$ mm.

Nevertheless, d_0 must be subtracted from the found distance $d_A = 19.97$ mm so that the new distance is

$$d_A = 19.97 - d_0$$

However, because d_0 is negative, the new distance d_A is actually longer: $d_A = 19.97 + 1.8 = 21.77$ mm.

This finally completes the matching design.

5.9 Excel Sheet (Slide 34)

This whole matching design procedure can be implemented by a succession of closed-form formulas, entered into Excel, for instance. A sample Excel object is included below. Double-click the table to open it. The data to enter are those in green-shaded fields: the frequency[2] and the magnitude and phase of the load reflection coefficient to be matched. If the length d_A is too short, you can increase it by setting the value of *n* from zero to 1 or more.

If one modifies the stub approximation coefficients, one can use the sheet for a different type of discontinuity introducing the parallel susceptance, e.g. a capacitive iris.

Speed of light	c_0	m/s	299792458
Waveguide broadside	wg_a	mm	86.36
Cutoff frequency	f_c	MHz	1735.71
Design frequency	f	MHz	2450
Free space wavelength	l 0	mm	122.36
Guide wavelength	l g	mm	173.38
Stub Approximation h_mm = f(S11.mag)			
Apparent S_{11} offset	d_0	mm	-1.8
Polynomial coefficients		Order	Value
Stub diameter: 20 mm		0	3.341328
Stub tip radius: 7 mm		1	44.277644
		2	-53.401810
		3	37.793248
Matching			
Load reflection coefficient magnitude	$\|\Gamma_L\|$		0.4845
Load reflection coefficient phase	φ_L	dg	201.9
Real part of reflection coefficient Γ_A	u_A		-0.235
Imaginary part of reflection coefficient Γ_A	v_A		0.424
Phase of reflection coefficient Γ_A	φ_A	dg	118.98
Phase shift from load plane T to stub plane A	Dφ	dg	82.92
Number of guide half-wavelengths to add	n		0
Distance from load plane T to stub plane A	d_A	mm	**21.77**
Tuning stub length (insertion)	h	mm	**16.56**

[2] Because the stub S_{11} is fairly frequency-independent, the same polynomial can be used for all frequencies in the ISM band.

5.10 Verification: Electromagnetic Simulation (Slide 35)

We have verified the matching design by means of electromagnetic simulation of the complete structure with the matching network parameters d_A = 21.77 mm, h = 16.56 mm.

We also simulated a structure with the stub shifted by a half of the guide wavelength at 2.45 GHz, i.e. with d_A = 21.77 + 86.69 = 108.46 mm.

Clearly, the ultimate verification should be by an experiment. Experience has shown that, for simple structures like these, measurement and simulation agree very well.

5.11 Electromagnetic Simulation (Slide 36)

The simulation shows that in both cases the curves of S_{11} are very close to optimal, being shifted down by 5 MHz.

It is also evident that a longer matching structure decreases bandwidth. The key lesson to be learned from this is: **Do your matching as close to the load as possible**.

Without doing anything more, the worst (highest) value of $|S_{11}|$ in the desired band 2.425 – 2.475 MHz is -19.2 dB for the shorter structure and -13.9 dB for the longer structure, which corresponds to the reflected powers of 1.2% and 4.1%, respectively.

However, the S_{11} curves can be moved to achieve better results. This can be accomplished by slight variations of the stub position along the waveguide (d_A) and the stub length (h). A most useful approach is to employ computer optimization. The real structure can ultimately be fine-tuned experimentally.

5.12 Optimization (Slide 37)

We carried out the optimization of the shorter structure using CST Studio Suite. Our goal was to minimize the maximal magnitude of S_{11} in the 2.425 – 2.475 GHz frequency range. The varied parameters were the stub distance d_A and the stub length h.

This optimization slightly improved the result, essentially centering the curve $|S_{11}(f)|$ at 2.45 GHz. The parameters have changed as follows:

- d_A changed from 21.77 mm by -0.06 mm to 21.71 mm
- h changed from 16.56 mm by -0.24 mm to 16.32 mm

The worst (highest) value of $|S_{11}|$ in the band 2.425 – 2.475 MHz is -20.5 dB, which corresponds to the worst-case reflected power of 0.9%.

5.13 Tolerance Analysis (Slide 38)

Tolerance analysis of critical parameters is an important step in any design process. We performed the tolerance analysis by varying the matching network parameters by ±0.2 mm as follows:

$d_A = 21.7 \pm 0.2$ mm; $h = 16.3 \pm 0.2$ mm

Thus, an ensemble of 9 curves has been obtained. The worst (highest) value of $|S_{11}|$ in the band 2.425 – 2.475 MHz is -18.9 dB, which corresponds to the reflected power of 1.3%.

5.14 Tuning by Stub Distance (Slide 39)

The subgroup of the tolerance analysis ensemble for the nominal stub length h = 16.3 mm and the stub distance d_A varying as 21.5 mm, 21.7 mm, 21.9 mm is a good indication of the fine-tuning possibilities by means of moving the stub along the waveguide. The plot shows that the result is essentially frequency-shifting the traces without significantly affecting their vertical position.

5.15 Tuning by Stub Length (Slide 40)

The subgroup of the tolerance analysis ensemble for the nominal stub distance d_A = 21.7 mm and the stub length h: varying as 16.1 mm, 16.3 mm, 16.5 mm is a good indication of the fine-tuning possibilities by means of changing the stub length. The plot shows that the stub length affects both the position of the S_{11} curve on the frequency axis as well as the value of S_{11} magnitude.

5.16 Parallel Susceptance "Providers" (Slide 41)

There are a variety of waveguide discontinuities that can provide the parallel susceptance suitable for impedance matching purpose. Among them, the tuning stub and the waveguide iris are the most common.

5.16.1 Tuning Stub

The tuning stub can be considered a "matching superstar". This is because

- it is simple,
- it is nearly frequency-independent,
- it can handle high powers,
- fine-tuning is easy.

The tuning stub is not suitable for providing very high parallel susceptances *b*, needed for matching of highly reflective loads. This is because of the extremely high sensitivity $\Delta b/\Delta h$ in the area of high *b*, which precludes reliable control of *b*.

An example of a case where the tuning stub is not appropriate is establishing a very weak coupling to a resonator loaded by a low-loss sample (such as sintering ceramic powders). In this case, a coupling iris is a better choice.

5.16.2 Waveguide Iris

The waveguide iris is another common choice. Its advantages are:

- It can provide either capacitive or inductive parallel susceptance.
- It is flat, and hence needs less axial space than the stub.
- It can provide readily controllable very high susceptances needed for weak coupling to high-quality resonator-type applicators. The iris is then implemented as a small hole in the waveguide partition.

The iris can also be a viable alternative in situations when the stub cannot be used, such as in circular waveguides with axially symmetric TM01 mode, which is an important mode in high-power microwaves.

A disadvantage of irises is difficult fine-tuning, in particular moving an iris along the waveguide.

With regard to the design of iris-based matching networks, the iris can be treated in exactly the same way as we have shown for the stub.

6. Automatic Impedance Matching

6.1 Need for Automatic Impedance Matching (Slide 43)

Reasons for considering automatic impedance matching include, among others:

- Saving of energy in production plants, where:
  - the installed microwave power may be on the order of megawatts
  - production runs 24 hours a day, 7 days a week
  - even a 5% of energy saving is significant
- Increased production rate because of shorter processing time.
- Improved quality of production in some applications, e.g. due to better stability of plasma.
- Extremely rapid volumetric heating is needed in some applications (e.g. killing of bacteria), where each watt spent in the load is important.

6.2 Practical Microwave Installations (Slide 44)

6.2.1 Generator

In practical high-power microwave installations, the generator is typically a magnetron head with a circulator. Therefore, we can assume that the generators are matched: $\Gamma_G \approx 0$.

6.2.2 Load

The load is typically an applicator with an object to be processed. In practice, applicators often reflect more than 50% of the incident power ($|\Gamma_L| \geq 0.7$). The main reasons for this include:

- Applicators are, in general, difficult to design precisely
- Object properties can vary in the course of the processing (drying, thawing, sintering, …)
- Different object types can be used in the same applicator
- Objects are moving (e.g. on a conveyor belt)
- Applicators are sometimes incorrectly designed

In situations like these, the Autotuner comes to the rescue.

6.3 Traditional Adjustable Matching Devices (Slide 45)

The adjustable matching devices most commonly used in the high-power microwaves are:

- Magic Tee
- Three-stub waveguide impedance transformer

6.4 Magic Tee (Slide 46)

The Magic Tee is a connection of four waveguides where the straight branch connects the generator with the load, and the two sidearms are terminated in sliding shorts.

Electrically, the device behaves as a simple L-type reactive two-port, with the reactances adjusted by the short positions. Reactances of both polarities can be realized. The network of this type can theoretically match any load at a single frequency.

The Magic Tee has several disadvantages:

- The sliding shorts can be either contacting or noncontacting.
  - Contacting shorts suffer from wear of the contacting fingers, unreliable contact, hazard of burning.
  - Noncontacting shorts (if poorly designed) can suffer from radiation leakage and arcing.
- Long travel of the shorts necessary to realize all reactances. The travel needed is a half of the guide wavelength ($\lambda_g/2$), which amounts to about 90 mm for WR340 waveguide and 230 mm for WR975 waveguide. The result is slow tuning process.
- Strong frequency dependence of the realized reactances since they are implemented by sections of transmission lines. This leads to a narrow matching bandwidth.
- The device is rather unwieldy, bulky and massive.

6.5 Three-Stub Impedance Transformer (Slide 47)

The three-stub (or triple-stub) impedance transformer consists of three manually or stepper motor-driven metallic cylinders (tuning stubs) inserted to variable extensions into the interior of a rectangular waveguide from the center of its broader wall. Electrically, a stub acts essentially as a capacitive shunt susceptance increasing with its extension. The mutual stub distances are equal to a quarter of guide wavelength λ_g at the maximal operating frequency (design frequency) f_d.

At one time, only two of the stubs are used for matching: either the first and second stubs (**1+2**) or the second and third stubs (**2+3**).

For automatic matching, the stubs must be accurately electrically characterized.

6.6 Two-Stub Tuning Principle (Slide 48)

Suppose the load reflection coefficient Γ_L (green circle) is referred to the plane of the stub 2 axis. Inserting stub 2 adds a positive parallel susceptance, moving Γ_L along a circle of constant real part of the normalized admittance until the bottom half of the circle -K_1 is reached. The resulting reflection coefficient (yellow circle) is now to be rotated clockwise nominally by 180° (this corresponds to the nominal λ_g/4 stubs separation) to obtain the reflection coefficient observed toward load from the plane of stub 1 axis. This reflection coefficient (red circle) lies on the upper half of the circle K_1 (that is why we wanted to move Γ_L to the bottom half of the circle -K_1). The circle K_1 represents admittances with a unit real part; the upper half of K_1 those with negative imaginary part. This imaginary part is now to be compensated by the parallel susceptance introduced by stub 1, which results in the desired match (gray circle).

Only the shaded portion of the Smith chart can be matched by stubs 1+2. To match the remaining part needs shifting of both stubs by λ_g/4 – or, in other words, using stubs 2+3 for the matching.

6.7 Tuner Electric Model (Slide 49)

The tuner equivalent circuit consists of three blocks representing the individual stubs, the blocks being separated by sections of transmission lines. An additional line section is between stub 3 and the load reference plane. Similarly, a line section can be present between stub 1 and the tuner input reference plane.

Each stub can be represented by a T-network with the series reactance x and the parallel susceptance b depending on the stub insertion h and frequency f.

The equivalent circuit is needed in autotuners for transforming the load reflection coefficient Γ_L to the tuner input reflection coefficient Γ_1 and vice versa. This enables *predictive* tuning algorithms, i.e. algorithms that can find the stub extensions needed for matching in a purely mathematical way.

6.8 Three-Stub Autotuner (Slide 50)

A three-stub autotuner consists of three main parts:

- Motorized tuner
- Vector reflectometer
- Internal computer

The motorized tuner usually employs stepper motors and needs specific motor driver electronic circuitry.

The "vector reflectometer" part is usually more than a mere reflectometer because it actually measures more quantities than the reflection coefficient, including:

  - Complex input reflection coefficient Γ_1
  - Incident power P_i
  - Frequency f
  - Internal temperature T

The internal computer stores all data needed for accurate measurement (calibration data), controls the whole measurement and matching process, and communicates with external controllers or monitoring devices.

6.9 Autotuning Procedure (Slide 51)

The automatic matching procedure consists of repeating the following basic steps, continuously or by external commands:

1. With the tuning stubs in the current extensions, measure the tuner input complex reflection coefficient Γ_1. For this, also the actual frequency must be measured.

2. From a known Γ_1, compute the load reflection coefficient Γ_L. Use the tuner electric model for the actual frequency and the current stub extensions h_1, h_2, h_3.
3. From a known Γ_L, determine new stub extensions to minimize the magnitude $|\Gamma_1|$ of the tuner input reflection coefficient. The matching condition (21) can be conveniently used for this purpose, i.e. $S_{22} = \Gamma_L^*$ where S_{22} refers to the three-stub tuner part of the autotuner.
4. Move the stubs to the new extensions.

6.10 Basic Autotuner Parameters (Slide 52)

The basic autotuner parameters include:

- Matching range
- Matching accuracy
- Matching speed
- Maximal working power

6.11 Matching Range (Slide 53)

The matching range of an autotuner is the area of the load reflection coefficients Γ_L that can be perfectly matched. The matching range can be expressed as:

- The single figure R_{max}, which is the maximal magnitude $|\Gamma_L|$ of the load reflection coefficient that can be matched regardless of the phase of Γ_L.
- The matchable area, which is a graphically outlined area in the complex plane of Γ_L showing all load reflection coefficients that can be perfectly matched. This form is more detailed than merely stating R_{max}.

6.12 Matchable Area (Slide 54)

The matchable area of a three-stub tuner consists of two sub-areas, one matchable by the stubs couple 1 + 2, the other matchable by the stubs couple 2 + 3.

The load reflection coefficients outside of the matchable area cannot be perfectly matched; and yet, the matching will still be improved.

The matchable area is frequency-dependent:

- For frequencies lower than a certain *design frequency* f_d, the two sub-areas overlap.
- At the design frequency, the sub-areas just touch.
- For frequencies higher than f_d, a gap opens between the two sub-areas.

The design frequency f_d should therefore be the *highest* working frequency of the tuner.

6.13 Matching Accuracy (Slide 55)

The matching accuracy can be quantified as the residual input reflection coefficient magnitude $|\Gamma_1|$ after setting the tuning stubs to the determined extensions.

The matching accuracy depends on:

- Measurement accuracy
- Tuner model accuracy

The consequences of inaccuracy can be:

- Imperfect tuning
- Fine-tuning taking several steps
- The system becoming unstable (oscillating stubs)

The first two effects affect the tuner overall performance only insignificantly.

The stub oscillations rarely occur, and only for certain load reflection coefficients. They usually cease after a small change of Γ_L, which takes place regularly in the course of an industrial process.

6.14 Matching Speed (Slide 56)

The matching speed can be quantified as the time elapsed between the start of a tuning cycle and an instant an acceptable match has been reached.

The matching speed depends mainly on two factors:

- Measurement time

- Stub travel time

6.14.1 Measurement Time

The measurement time typically ranges from 100 μs to 1 s. It is sometimes dictated by circumstances. For instance, there may be a need to average out periodic variations of the load reflection coefficient caused by mode stirrers or objects moving in an applicator. A proper measurement (sampling) time is in this case one or more full periods of these variations.

6.14.2 Stub Travel Time

The stub travel time depends on the used motors and the full extension length of the stubs, which in turn is dictated by the waveguide size.

The times needed for the full stub travel range typically between 0. 5 s and 5 s. However, stubs do not travel the full span often: only a fraction of it is necessary for small corrections needed in practical situations.

6.14.3 Tuning Cadence

A typical tuning cadence for most of the situations is 5 cycles/s.

6.15 Maximal Working Power (Slide 57)

The maximal working power of a three-stub tuner is mainly limited by three factors:

1. Electric breakdown which, if unattended, can lead to arcing and tuner destruction.
2. Stubs overheating due to high surface current density.
3. Radiation leakage past the noncontacting stubs. The leaked radiation may interfere with the neighboring electronics, or even present a health hazard.

Of the three, the electric breakdown and arcing is the most frequent and the most severe occurrence.

6.16 Electric Breakdown and Arcing (Slide 58)

Matching high-reflection loads (i.e. loads with very small losses) requires extending one or two tuning stubs to near-maximum positions. The space between the stubs and the load then acts as a high-quality resonator. At certain circumstances, brought about by an unfortunate combination of frequency, load reflection coefficient and tuner-load distance, a resonance can take place. Field strengths in such a resonating structure may exceed the intensities in a matched system many times over. High electric field strength can lead to electric breakdown that can develop in arc.

Consequently, more than being a tuner internal property, this is a matter of physics and considerate use of the tuner.

6.17 Electric Discharge and Arcing (Slide 59)

The critical load reflection coefficients, for which the resonance can occur, are those closest to the unit circle. They are nearly totally reflecting, and yet can be perfectly matched. This means that the whole generator power is being "pumped" into a space which does not absorb. Consequently, something must yield. Not surprisingly, the highest field concentration occurs at the bottom edge of one of the extended tuning stubs. The arc that can develop starts absorbing the generator power and is able to melt the stubs within a short time.

6.18 What Can Be Done (Slide 60)

Several precautions can be made to prevent arcing or limit the damages it can cause. The possibilities include:

1. One should design the applicator such that a given magnitude of the load reflection coefficient is never exceeded (e.g. $|\Gamma_L| < 0.95$). This is, however, often easier said than done.
2. Install arc detectors and switch off the generator whenever arcing is detected.
3. Reduce the maximal stub insertion depth. The undesired consequence is that the whole matchable area will shrink.
4. Truncate the matching radius to a certain maximal value $R_{\max}$. If the load reflection coefficient magnitude exceeds $R_{\max}$, the input to the tuning algorithm will be the reflection coefficient Γ_M with the same phase as Γ_L but the magnitude equal to $R_{\max}$. Thus, the tuning stubs will never reach hazardous extension combinations.

6.19 Radiation Leakage (Slide 61)

A noncontacting tuning stub is a stub that glides in a dielectric sleeve or is kept in axial position by dielectric rings. This prevents the direct galvanic contact of the stub and the tuner body. A downside of such an arrangement is that the dielectric gap surrounding the stub is a way of escape of stray radiation. To block this radiation, a structure called a microwave choke must be incorporated in the leakage path. The choke is essentially a simple bandstop or "notch" filter, implemented as a series-connected, short-circuited quarter-wavelength section of a transmission line. At the corresponding frequency, such a line introduces an infinite impedance in series with the signal path, blocking thus the radiation from escaping.

6.20 Isolation Measurement (Slide 62)

The diagram shows, by illustration, the results of a choke isolation measurement. The isolation is quantified as a transmission coefficient from the waveguide port 1 to the coaxial port 2, as designated in the previous slide. Measurements of this kind are usually performed with the second waveguide port (denoted *Load* in the previous slide) terminated in a match. The result is the red trace. However, leakage can be up to 12 dB higher if the load differs from the match: the blue trace represents the worst case.

Thus, in our illustration, the worst-case isolation in the 2.425 – 2.475 GHz is about -36 dB. This means, for instance, that if the power of the forward wave in the waveguide is 20 kW, the leaked power can be as high as 5 W, a bothersome level.

As the illustration demonstrates, chokes are relatively narrowband devices, and therefore must be carefully tuned (the choke in our example could also be tuned better). Unlike the issue of arcing, which, to a large degree can be avoided by the user, tuning a choke is the manufacturer's responsibility.

7. Optimal Matching

7.1 Theory Is Useful… (Slide 64)

The theory of the impedance matching assumes that the input to tuning algorithms is a *single complex number* – load reflection coefficient Γ_L [3]. The tuning algorithm causes Γ_L to be transformed to $\Gamma_1 = 0$ at the matching network input.

But real life is not so simple…

7.2 …Life Is Not So Simple (Slide 65)

Two main factors complicate the situation:

- Powering magnetrons with high-ripple high-voltage supplies
- An oscillating load reflection coefficient

7.2.1 High-Ripple Power Supplies

High-ripple magnetron power supplies produce anode voltage and current that pulsate with frequency f_L of the power line (50 Hz or 60 Hz) or, depending on the used rectification scheme, its multiples up to $12 \times f_L$ (600 Hs or 720 Hz). In this cadence, also the generated power and frequency are modulated. For single-phase rectifiers, the power even drops to zero at portions of the period.

7.2.2 Oscillating Load Reflection Coefficient

The applicator reflection coefficient Γ_L often oscillates, most often due to the presence of mode stirrers or objects moving within the applicator, for instance on a conveyor belt. These variations can affect the magnetron frequency and the generated power, often even if a circulator is present. Typical period of such variations is 0.1 – 1 s.

7.2.3 How to Handle

Oscillations of both types are too fast for an autotuner to handle in real time, and usually this would not even be reasonable. Therefore, some sort of averaging is needed. The averaging should be such as to lead to the best possible match given the existing signal waveforms and given the autotuner capabilities.

---

[3] Of course, the tuning algorithm also needs to know the frequency, the current stub positions, and the tuner electric model, e.g. its scattering parameters $S_{ij}(f, h_1, h_2, h_3)$.

7.3 Signal Waveforms (Slide 66)

This slide presents an example of fluctuating data. It is a consequence of bottles being propelled through a small applicator excited by a magnetron generator fed from a high-ripple anode voltage supply. The plot of the load reflection coefficient magnitude $|\Gamma_L|$ shows the magnitude oscillating between 0.25 and 0.75, with period 800 ms. The polar chart reveals that the contour of Γ_L forms a half-moon pattern.

The generated power P_G exhibits slow modulation of the form and period following those of $|\Gamma_L|$. This proves that the magnetron can indeed "feel" the load variations in spite of the circulator presence.

The generated power P_G, however, also exhibits higher-frequency ripples, those with period 20 ms. Origin of these ripples is the unfiltered three-phase rectification scheme used. The combination of Δ-Y voltages produces components with frequency as high as 12 × 50 = 720 Hz in the waveform, as the zoomed P_G plot discloses.

The mean generator (incident) power is 36 kW; the mean reflected power is 13 kW, which results in the energy efficiency η = 65%. The optimal matching task is to decrease the mean reflected power as much as possible.

The shown traces were obtained by sampling the signals with rate 800 Hz (sample separation 1.25 ms) over the interval T_s equal to two periods of the slowest oscillations, i.e. $T_s = 2 \times 800$ ms = 1600 ms.

7.4 Optimal Matching (Slide 67)

Optimal matching means handling the fluctuating signals and setting up the autotuner in such a way as to arrive at the highest possible *mean* power delivered to load. Optimal matching consists of two steps:

1. Acquire data (Γ_L, P_G) by proper sampling the signals (responsibility of user).
2. Process the samples optimally (responsibility of autotuner).

7.4.1 Acquiring Data

These are basic rules for obtaining an appropriate ensemble of samples for further processing:

- Determine your signal waveforms (Γ_L, P_G).
- Set sampling *rate* high enough to capture the fastest signal ripples.
- Set sampling *duration* equal to an integral multiple of the longest period detected in the signal.

Since acquiring data appropriately is the responsibility of the user, this may be a stumbling block for unskilled personnel.

7.4.2 Processing Samples

Processing the acquired samples $\Gamma_{L,n}$ and $P_{G,n}$, $n = 1, 2, \ldots N$, optimally means finding such a single reflection coefficient value Γ_B such that, when used as an input to the tuning algorithm, it will result in the highest possible mean power delivered to the load[4].

It can be shown that averaging the reflection coefficient samples weighted by the associated generator power samples results in a value of Γ_B that very closely meets the above criterion. Weighting by power means that the samples of Γ_L taken while power P_G is higher have more importance than low-power samples. Geometrically, Γ_B can be viewed as the "center of mass" of the Γ_L pattern displayed in the polar chart.

In the illustrative example, Γ_L oscillates along a circle. The samples displayed as the filled black squares correspond to higher generator powers P_G; the empty squares correspond to lower generator powers. Therefore, the weighted average Γ_B is not at the circle center but closer to the high-power samples.

The obtained Γ_B will then be used for tuning as a "deputy" for all the involved samples. The tuning algorithm will find tuning stubs extensions that would transform Γ_B to zero. While Γ_B is an abstraction, the tuner with these stub extensions will transform each individual sample to lie on the circle Γ_1, which, although having larger radius, is located closer to the origin. Consequently, the mean reflected power will be smaller, actually the smallest possible. Optimal matching for this case has been accomplished.

[4] For a matched generator, this requirement is identical with minimizing the mean reflected power.

7.5 Optimal Matching Example (Slide 68)

The effect of the outlined optimal matching procedure on our example is shown in this slide. The red "half-moon" pattern in the top left diagram represents the samples of load reflection coefficient Γ_L. The "center of mass" of this pattern is Γ_B. The tuning would force Γ_B to the origin at the tuner input. As a result, it transforms the red Γ_L pattern to the blue Γ_1 pattern, which is closer to the origin. The difference is obvious in the magnitude plot (top right) and in the reflected power plot (bottom right). The mean reflected power has been reduced from 13 kW to 5 kW, increasing energy efficiency by 20% to the resulting $\eta = 85\%$.

8. Stub Swapping Problem

8.1 Stub Swapping (Slide 70)

Only two of the three stubs (1+2 or 2+3) are used at a time for matching of any particular load reflection coefficient, each pair covering a given sub-area of the Smith chart, the sub-areas designated A12 and A23, respectively. There is a boundary between these two sub-areas (the thick red line) where the central stub (stub 2) is withdrawn and either stub 1 or stub 3 protrudes (up to maximum). Varying the load reflection coefficient just slightly across this border would force the stubs 1 and 3 to incessantly and mostly uselessly jump up and down. Because of this stub switching, the boundary can be aptly termed the "discontinuous boundary" [5].

8.2 Undesirable Consequences (Slide 71)

Undesirable consequences of incessant jumping of stubs 1 and 3 are:

- Reduced autotuner lifespan.
- Input match is not under control during the transient process of the stub swapping. If the stubs move between the starting and the final positions with full speed, high transient peaks of the reflected power can occur. The consequences are:
  - Waste of energy.
  - The transients may trip the magnetron protection. This, in turn, may disrupts the industrial process.
  - In plasma applications, the transients can cause plasma instability.

8.3 Countermeasures (Slide 72)

The currently existing countermeasures only reduce the occurrence of stub swapping but do not affect the transient process itself. These countermeasures include:

- Introducing a hysteresis
- Phase-shifting the load reflection coefficient

8.3.1 Hysteresis

The concept of hysteresis aims at reducing the occurrence of stub swapping. The principle lies in introducing a guarding area H around the swapping boundary. The swapping will not occur as long as the load reflection coefficient Γ_L remains within the guarding area. The price is somewhat greater mismatch, especially if the width of the guarding area is chosen too high.

8.3.2 Phase-Shifting Load Reflection Coefficient

Phase-shifting the load reflection coefficient involves inserting a section of waveguide between the load and the tuner input. If the waveguide length is $\lambda_g/8$, where λ_g is the guide wavelength, then the load reflection coefficient perceived by the tuner will be rotated 90° clockwise relative to the original Γ_L. This will remove it from the swapping area to a "safe" swapless location. The drawbacks of this approach are that the implementation is cumbersome, and that this method is load-specific: if the load reflection coefficient changes, the situation can go back to being bad.

[5] Another, less critical, is the "continuous" boundary (the blue line), where the central stub is inserted and stubs 1 and 3 swap "gracefully" by way of going through zero extensions.

8.4 Controlled Stub Motion (Slide 73)

A theoretically investigated but not yet practically implemented method of alleviating the stub swapping consequences is a controlled stub movement, affecting the transient process itself. Instead of driving the stubs to their destination extensions at full speed each, the stubs are extended in a controlled, mutually dependent manner, keeping the input reflection coefficient excursion to a minimum. A drawback could be that the process takes longer than the full-speed approach.

9. Autotuner Example

9.1 Inside HOMER Autotuner (Slide 76)

The HOMER autotuner consists of the measurement instrument, called Analyzer, and a motorized tuner, called the Mototuner.

The Analyzer part of the system is based on the six-port reflectometer (SPR) principle. SPR is suitable for high-power industrial applications due to its capability of precision on-line measurement of

- complex (vector) reflection coefficient of microwave loads,
- incident, reflected and absorbed powers

all of this at full working-power conditions of industrial installations. Moreover, the complex reflection coefficient can be used in predictive impedance matching algorithms actuating the Mototuner, thus enabling the function of fast, compact Autotuners.

The principal microwave components of the SPR are three voltage probes (antennas P1, P2, P3) and a reference coupler. The insertion depth h_{ant} of the probes and the coupling factor of the reference coupler govern the nominal working power of the system.

Microwave signals from the probes and the reference coupler are converted to DC in amplitude detectors, then amplified and converted to digital in A/D converters (ADC) of a multifunction PC-card SD-2000. Solid-state switches preceding the detectors serve for measuring offsets of the DC circuitry (apparent signals in absence of microwave power) to be subtracted from the measured voltages V_1 to V_4. The switches are controlled by a digital input/output (DIO) portion of the SD-2000. In a single-board computer (SBC), connected with SD-2000 via PC/104 interface, the digitized voltages undergo a correction routine which eliminates nonlinearity and temperature dependence of the detector transfer characteristics. The detector temperature is measured by a temperature sensor, connected to the SD-2000. The corrected voltages along with the signal frequency and the stored SPR calibration data are used to compute the reflection coefficient Γ_L and the incident power $P_i = P_G$.

The knowledge of signal frequency is a precondition for accurate measurement. Since magnetrons used in industrial applications are free-running sources with frequency depending on working conditions, frequency is not known in advance and must be measured. A frequency counter is therefore integrated with the system. The counter uses a separate directional coupler (Frequency Coupler) of basically the same design as the reference coupler. The coupled signal frequency is divided in a microwave prescaler down to about 50 – 100 MHz, which is then measured by a 150-MHz counter (CNT) implemented in SD-2000.

The system communicates with an external controller (e.g. a PC) via RS232 or CAN Bus interface. Another function implemented in the SD-2000 is the control of the three Mototuner stepper motors.

Residing inside the SBC is the HOMER firmware, which is a system of files that control its operation and behavior. The firmware consists of an executable file (Server), calibration files (containing individual characteristics of the system) and user-definable configuration files, defining HOMER behavior.

10. Literature

10.1 Some Great Books (Slide 80)

[1] Matthaei, G.L., Young, L., Jones, E.M.T., *Microwave Filters, Impedance Matching Networks, and Coupling Structures*, McGraw-Hill, New York, 1964. (aka “Filter Bible”.)

[2] Pozar, D. M., *Microwave Engineering – 4th Ed.*, Wiley & Sons, New York, 2012. (Classic textbook. Matching: Chapter 5.)

[3] Engen, G. F., Microwave Circuit Theory and Foundations of Microwave Metrology, Peter Peregrinus, London, 1992. (Essentials of microwave circuits.)